ED-460

M.A./M.Sc. 2nd Semester
Examination, May-June 2021

MATHEMATICS

Paper - II

Real Analysis-II

Time : Three Hours] [*Maximum Marks* : 80

Note : Answer any **two** parts from each question. All questions carry equal marks.

Unit-I

1. (*a*) Let f is bounded on $[a, b]$, f has only finitely many points of discontinuity on $[a, b]$ and α is continuous at every point at which f is discontinuous. Then show that $f \in \mathbb{R}(\alpha)$.

(*b*) Let α be a monotonically increasing function on $[a, b]$ and $\alpha' \in \mathbb{R}\,[a, b]$. Let f is bounded real function on $[a, b]$. Then prove that $f \in \mathbb{R}(\alpha)$ if and only if $f\alpha' \in \mathbb{R}\,[a, b]$. In that case

$$\int_a^b f d\alpha = \int_a^b f(x)\alpha'(x)\,dx .$$

(*c*) If γ' is continuous on $[a, b]$, then prove that γ is rectifiable and $\Lambda_\gamma = \int_a^b |\gamma'(t)|\,dt$.

Unit-II

2. (*a*) Prove that a countable union of measurable set is a measurable set.

(*b*) Show that a function is simple iff it is measurable and assumes only a finite number of values.

(*c*) Let $\{E_i\}$ be an infinite decreasing sequence of measurable sets; that is, a sequence with $E_{i+1} \subset E_i$, for each $i \in N$. Let $m(E_i) < \infty$ for at least one $i \in N$. Then show that $m\left(\bigcap_{i=1}^{\infty} E_i\right) = \lim_{n\to\infty} m(E_n)$.

Unit-III

3. (*a*) The class $\mathscr{B}$ of all μ^*-measurable sets is a σ-algebra of subsets of X. If $\bar{\mu}$ is μ^* restricted to $\mathscr{B}$, then prove that $\bar{\mu}$ is a complete measure on $\mathscr{B}$.

(*b*) Show that the set function μ^* is an outer measure.

(*c*) Let (X, S, μ) be a σ-finite measure space, Σ a semi ring of sets such that $S \subset \Sigma \subset \mathscr{B}$, and $\bar{\mu}$ is a measure on Σ. If $\bar{\mu} = \mu$ on S, then prove that If $\bar{\mu} = \mu^*$ on Σ.

Unit-IV

4. (*a*) Evaluate the four derivatives at $x = 0$ of the function given by

$$f(x) = \begin{cases} ax.\sin^2\left(\frac{1}{2}\right) + bx.\cos^2\left[\frac{1}{x}\right] & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ a'x.\sin^2\left(\frac{1}{x}\right) + b'x.\cos^2\left[\frac{1}{x}\right] & \text{if } x < 0 \end{cases}$$

where $a < b$ and $a' < b'$.

(*b*) State and prove Vitali's covering theorem.

(*c*) Let f be a bounded and measurable function defined on $[a, b]$ if

$$F(x) = \int_a^x f(t)\,dt + F(a)$$

then $F'(x) = f(x)$ *a.e.* in $[a, b]$.

Unit-V

5. (*a*) Prove that the L^p spaces are complete.

(*b*) State and prove Riesz theorem.

(*c*) State and prove Egoroff's theorem.