ED-461

M.A./M.Sc. 2nd Semester
Examination, May-June 2021

MATHEMATICS

Paper - III

General and Algebraic Topology

Time : Three Hours] [*Maximum Marks* : 80

Note : Answer any **two** parts from each question. All questions carry equal marks.

Unit-I

1. (*a*) Define Projection maps with an example. Prove that if (X_1, T_1) and (X_2, T_2) are any two topological spaces then the collection $\beta = \{G_1 \times G_2 : G_1 \in T_1, G_2 \in T_2\}$ is a base for the same topology on $X = X_1 \times X_2$.

(*b*) Define product topology. Prove that the projection functions are open.

(*c*) Explain evaluation function with an example of real life. If f_i be a mapping of a topological space Y_j, $\forall\ i \in I$, then prove that the evaluation map e of X into $\pi\ \{Y_i : i \in I\}$ is continuous iff f_i is continuous for all $i \in I$.

Unit-II

2. (*a*) Define compactness with an example. Prove that a product space $X = T_1\ \{X_i : i \in I\}$ is a T_1-space iff each coordinate space is T_1-space.

(*b*) Define connectedness with an example. State and prove Tychnoff's theorem.

(*c*) Explain metrizable spaces. Prove that metrisability is countably productive property.

Unit-III

3. (*a*) Explain paracompactness. State and prove the Urysohn metrization theorem.

(*b*) Explain embedding. Prove that every regular, Lindeloff space is paracompact.

(*c*) Give an example of Hausdorff space. Let X be a regular space with a basis B that is countably locally finite. Then prove that X is normal and every closed set in X is a G_δ set in X.

Unit-IV

4. (*a*) Define Filter with an example. Prove that a topological space (X, T) is Hausdorff iff every net in X can converge to at most one point.

(*b*) Define directed set with an example. Prove that a filter F on a set X is an ultra filter iff for any two subsets A, B of X such that $A \cap B \in F$, we have either $A \in F$ or $B \in F$.

(*c*) Explain convergence of Net. Prove that a filter F on X is an ultrafilter iff F contains all those subsets of X which intersect every member of F.

Unit-V

5. (*a*) Explain covering spaces. Let f_1, f_2, g_1, g_2 be paths such that $f_1 \sim g_1$, and $f_2 \sim g_2$. Then prove that if $f_1 * f_2$ exists then $g_1 * g_2$ exists and $f_1 * f_2 \sim g_1 * g_2$.

(*b*) Explain group isomorphism with an example. If $f : X \to Y$ is continuous map, then prove that there exists a homomorphism $f^* : \pi_1 (X, x_0) \to \pi_1 (Y_1, f(x_0))$, where x_0 is any point in X.

(*c*) Define the fundamental group with an example. Prove the Fundamental theorem of Algebra.

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