ED-462

M.A./M.Sc. 2nd Semester
Examination, May-June 2021

MATHEMATICS

Paper - IV

Advanced Complex Analysis-II

Time : Three Hours] [*Maximum Marks* : 80

Note : Answer any **two** parts from each question. All questions carry equal marks.

Unit-I

1. (*a*) State and prove Weierstrass factorization theorem.

(*b*) To prove for $\operatorname{Re} z > 1$,

$$G(z)\overline{|(z)} = \int_0^\infty \left(e^t - 1\right)^{-1} t^{z-1} dt$$

(*c*) State and prove Mittag-Leffler's theorem.

Unit-II

2. (*a*) State and prove Schwarz reflection principle.

(*b*) Show that the series $\sum_{n=0}^{\infty} \frac{z^n}{z^{n+1}}$ and $\sum_{n=0}^{\infty} \frac{(z-i)^n}{(z-i)^{n+1}}$ are analytic continuation to each other.

(*c*) Let $\gamma : [0, 1] \to C$ be a path and let $\{(f_t, D_t) : 0 \le t \le 1\}$ be an analytic continuation along γ. For $0 \le t \le 1$ let $R(t)$ be the radius of convergence of the power series expansion of f_t about $z = \gamma(t)$. Then either $R(t) = \infty$ or $R : [0, 1] \to (0, \infty)$ is continuous.

Unit-III

3. (*a*) State and prove Harnack's theorem.

(*b*) Let G be a region and $f : \delta_{\infty} G \to R$ a continuous function. Then show that $u(z) = \sup \{\phi(z) : \phi \in P(f, G)\}$ defines a harmonic function u in G.

(*c*) Let G be a region, then prove that :

(*i*) The matrix space $H(G)$ is complete.

(*ii*) If $\{u_n\}$ is a sequence in $H(G)$ such that $u_1 \leq u_2 \leq$ ..... then either $u_n(z) \to \infty$ uniformly on compact subset of G or $\{u_n\}$ converges in $H(G)$ to a harmonic function.

Unit-IV

4. (*a*) Define order of an entire function. Find the order of polynomial $P(z) = a_0 + a_1 z + a_2 z^2 + ... + a_n z^n, \ a_n \neq 0$.

(*b*) State and prove Poisson-Jensen formula.

(*c*) State and prove Borel's theorem.

Unit-V

5. (*a*) State and prove Bloch's theorem.

(*b*) State and prove Schottky's theorem.

(*c*) State and prove $\frac{1}{4}$-theorem.
