ED-463

M.A./M.Sc. 2nd Semester
Examination, May-June 2021

MATHEMATICS

Paper - V

Advanced Discrete Mathematics-II

Time : Three Hours] [*Maximum Marks* : 80

Note : Answer any **two** parts from each question. All questions carry equal marks.

Unit-I

1. (*a*) Define connectivity of a graph and prove that if the intersection of two paths in a graph is a disconnected graph then the union of the two paths has at least one circuit.

(*b*) Define Tree and prove that a graph is a tree if and only if there is one and only path between every pair of vertices.

(*c*) Define planar graph and state and prove Euler's formula for connected planar graph.

Unit-II

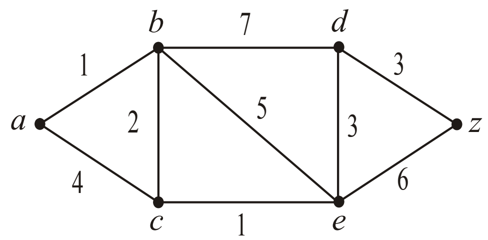
2. (*a*) Define fundamental cut sets and prove that every circuit has an even number of edges in common with every cut set.

(*b*) Explain the incidence matrix and adjacency matrix of a graph.

(*c*) The necessary and sufficient condition for a connected graph *G* to be an Euler graph is that 'all vertices of *G* are of even degree'. Show that.

Unit-III

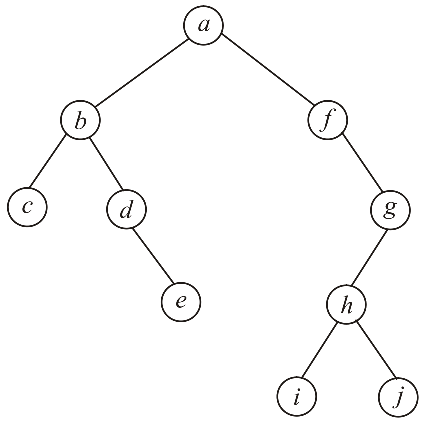
3. (*a*) Define weighted graph and write an algorithm for shortest path in weighted graph and use it to find shortest path from *a* to *z* in the graph shown in fig. where numbers associated with the edges are the weights.



(*b*) Explain Warshall's algorithm and lct $A = \{1, 2, 3, 4\}$ and $R = \{(1, 2), (2, 3)$ $(3, 4)$ $(2, 1)\}$ be a relation on R then find transitive closure of R.

(*c*) Explain Tree Traversals and determine the order in which the vertices of the binary tree given below will be visited under

(i) In order (ii) Pre order (iii) Post order



Unit-IV

4. (a) Design a finite state machine M which can add two binary numbers and compute the sum of 101110 and 010011.

(b) Define equivalent states and find π_0, π_1 and π_2 for the following finite state machines :

	State	Input		Output
		0	1	
⇒	S_0	S_1	S_5	0
	S_1	S_0	S_5	0
	S_2	S_6	S_0	0
	S_3	S_7	S_1	0
	S_4	S_0	S_6	0
	S_5	S_7	S_2	1
	S_6	S_0	S_3	1
	S_7	S_0	S_2	1

(*c*) Define homomorphism. Let S be any state in a finite state machine and let x and y be any words then $f(S, xy) = f(f(S, x), y)$ and $g(S, xy) = g(f(S, x), y)$.

Unit-V

5. (*a*) Define finite state automaton and design a finite state automaton that accepts those strings over {0, 1} such that the number of zeros is divisible by 3.

(*b*) Construct deterministic finite state automaton equivalent to the following non deterministic finite state automaton $M = (\{0, 1\}, \{S_0, S_1\}, S_0, \{S_1\}, f\}$ where f is given by the table

S \ I	f	
	0	1
S_0	$\{S_0, S_1\}$	$\{S_1\}$
S_1	ϕ	$\{S_0, S_1\}$

(*c*) Write any two differences between Moore and Mealy Machine and consider the Mealy Machine described by the transition tables. Construct a Moore Machine which is equivalent to the Mealy Machine.

	Present state	Input $a = 0$		Input $a = 1$	
		state	output	state	output
⇒	S_1	S_3	0	S_2	0
	S_2	S_1	1	S_4	0
	S_3	S_2	1	S_1	1
	S_4	S_4	1	S_3	0