ED-762

M.A./M.Sc. 4th Semester
Examination, May-June 2021

MATHEMATICS

Paper - I

Functional Analysis-II

Time : Three Hours] [*Maximum Marks* : 80

Note : Answer any **two** parts from each question. All questions carry equal marks.

Unit-I

1. (*a*) State and prove closed graph theorem.

(*b*) Let X be a Banach space and Y be a normed linear space. Let $\{T_i\}$ be a non-empty set of continuous linear transformation from X into Y, such that $\{T_i(x)\}$ is bounded for each x and X, then show that is $\{\|T_i\|\}$ is bounded.

(*c*) Let T be a bounded linear transformation from a Banach space X into a normed linear space Y. Then show that the openness of T implies the completness of Y.

Unit-II

2. (*a*) Let X and Y be normed linear space. Then show that $B(X, Y)$ the set of all bounded linear transformations from X into Y, is a normed linear space.

(*b*) Let X is a Banach space. Then show that X is reflexive if and only if X^* is reflexive, where X^* is the conjugate space of a normed linear space X.

(*c*) Let E be a real normed linear space and let M be a linear subspace of E. If $f \in M^*$, then show that there is a $g \in E^*$ such that $f \subset g$ and $\| g \| = \| f \|$.

Unit-III

3. (*a*) State and prove Bessel's inequality.

(*b*) If X is an inner product space and $x, y \in X$, then show that $|(x, y)| \leq ||x|| \, ||y||$.

(*c*) Show that a Banach space is a Hilbert space if and only if the parallelogram law holds.

Unit-IV

4. (*a*) State and prove Riesz Representation theorem.

(*b*) Prove that every Hilbert space is reflexive.

(*c*) Let T be an operator on a Hilbert space H. Then there exists a unique operator T^* on H such that

$$(Tx, y) = (x, T^*y)$$

for all $x,\ y \in H$.

Unit-V

5. (*a*) If T_1 and T_2 are self-adjoint, then show that $T_1 T_2$ is self-adjoint if and only if they commute, i.e. $T_1 T_2 = T_2 T_1$.

(*b*) State and prove generalized Lax-Milgram theorem.

(*c*) If T is a normal operator on a Hilbert space H and D is any scalar, then show that $T\text{-}\lambda I$ is also normal.

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