ED-763

M.A./M.Sc. 4th Semester
Examination, May-June 2021

MATHEMATICS

Paper - II

Partial Differential Equations and Mechanics

Time : Three Hours]   [*Maximum Marks* : 80

Note : Answer any **two** parts from each question. All questions carry equal marks.

Unit-I

1. (*a*) State and prove Hamilton ODE.

    (*b*) Derive Hopf-Lax formula.

    (*c*) For asymptotics in $\lfloor\infty$ norm, there exists a constant C such that $|u(x,t)| \le C/\sqrt{t}$.

Unit-II

2. (*a*) Use separation of variables to solve the porous medium equation

$$u_t = \Delta\left(u^{\Upsilon}\right) = 0 \quad \text{in} \quad \mathbb{R}^n \times (0, \infty).$$

(*b*) State and prove Plancherel's theorem.

(*c*) Derive Hopf-Cole transformation.

Unit-III

3. (*a*) Explain about vanishing viscosity method for Burger's equation.

(*b*) Write about asymptotics for linear terms.

(*c*) Define Majorants. Show that if

$$f = \sum_{\alpha} f_{\alpha} \cdot x^{\alpha} \quad \text{converges for} \quad |x| < r \text{ and}$$

$0 < s\sqrt{n} < r$ then f has a majorant for $|x| < s\sqrt{n}$.

Unit-IV

4. (*a*) State and prove the principle of least action.

(*b*) Explain about Poincare-Cartan integral.

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(3)

(*c*) Show that the transformation

$$p = q \, \cot p, \quad Q = \log\left(\frac{1}{q} \sin p\right)$$

is cannonical.

Unit-V

5. (*a*) State and prove the relation between Lagrange's and Poisson's brackets.

(*b*) Prove that the Poisson bracket of two constants of motion is itself a constant of the motion.

(*c*) State and prove Jacobi Identity through Poisson bracket.

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