



ED-771

M.A./M.Sc. 4th Semester
Examination, May-June 2021

MATHEMATICS

Optional - B

Paper - V

Graph Theory

Time : Three Hours] [*Maximum Marks* : 80

Note : Answer any **two** parts from each question. All questions carry equal marks.

1. (a) Show that for any two positive integers $s_1, s_2 \geq 2$ $R(s_1, s_2) \leq R(s_1 - 1, s_2) + R(s_1, s_2 - 1) \dots$ (1) and if both the terms on the right of (1) are even strict inequality obtains in (1).
- (b) Prove that for any $s \geq 2$. $R(s, s) \geq 2^{s/2}$.
- (c) Prove that $R_k(3) \leq \lfloor k!e \rfloor + 1$.

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(Turn Over)

(2)

2. (a) Prove that every group is isomorphic to the automorphism group of some graph.
- (b) Prove that an edge-transitive graph without isolated vertices is either vertex transitive or bipartite.
- (c) Prove that if the even values of the digraph D are all distinct then $T(D)$ is Abelian.
3. (a) Prove that for any edge e of a graph G ,
 $\phi(G, x) = \phi(G - e, x) - \phi(G | e, x)$.
- (b) Prove that the Tutte polynomial is the same as the dichromatic.
- (c) Prove that for any graph G , the chromatic polynomial
$$\phi(G, x) = (-1)^v x^k T(G, 1 - x, 0).$$
4. (a) Write a short note on graph enumeration.
- (b) State and prove Burnside's lemma weighted form.
- (c) Prove that $g_n(x) = Z(s_n^{[2]}, 1+x)$ where $s_n^{[2]}$, is the group of permutations on the object set $V_{(2)}$ of unordered pairs of elements of 'vertex' set of n elements induced by the symmetric group s_n acting on V .

(3)

5. (a) Prove that every digraph without odd cycles has a 1-basis.
- (b) Prove that a weak digraph is strong iff each of its blocks is strong.
- (c) Prove that the transportation network has a feasible iff

$$d(Y \cap \bar{S}) - s(X \cap \bar{S}) \leq c(S, \bar{S})$$

for every subset S of V .
