## ED-618

M.A./M.Sc. 3rd Semester Examination, March-April 2021

## MATHEMATICS

Optional - A
Paper - IV
Operations Research - I

Time : Three Hours] [Maximum Marks : 80
[Minimum Pass Marks : 16
Note : Answer any two parts from each question. All questions carry equal marks.

## Unit-I

1. (a) Use Simplex method to solve the following linear programming problem:

Maximize $\quad z=6 x_{1}+8 x_{2}$
Subject to : $5 x_{1}+10 x_{2} \leq 60$
$4 x_{1}+4 x_{2} \leq 40$
$x_{1}, x_{2} \geq 0$

## (2)

(b) Solve the following linear programming problem using the result of its dual:

Minimize $\quad z=24 x_{1}+30 x_{2}$
Subject to: $2 x_{1}+3 x_{2} \geq 10$
$4 x_{1}+9 x_{2} \geq 15$
$6 x_{1}+6 x_{2} \geq 20$
$x_{1}, x_{2} \geq 0$
(c) Consider the following linear programming problem:

Maxmize $\quad z=10 x_{1}+15 x_{2}+20 x_{3}$
Subject to: $2 x_{1}+4 x_{2}+6 x_{3} \leq 24$

$$
3 x_{1}+9 x_{2}+6 x_{3} \leq 30
$$

$x_{1}, x_{2}, x_{3} \geq 0$
and check whether the optimality is affected, if the profit coefficients are changed from $(10,15,20)$ to $(7,14,15)$. If so, find the revised optimum solution.

## Unit-II

2. (a) Solve the following linear programming problem using big-M method:

## (3)

Minimize $\quad z=2 x_{1}+3 x_{2}$
Subject to : $x_{1}+x_{2} \geq 6$

$$
7 x_{1}+x_{2} \geq 14
$$

$$
x_{1}, x_{2} \geq 0
$$

(b) Solve the following linear programming problem using dual simplex method:

Minimize $\quad z=2 x_{1}+4 x_{2}$
Subject to : $2 x_{1}+x_{2} \geq 4$

$$
x_{1}+2 x_{2} \geq 3
$$

$$
2 x_{1}+2 x_{2} \leq 12
$$

$$
x_{1}, x_{2} \geq 0
$$

(c) Find the Dual of the Primal:

Maximize $\quad z=x_{1}+5 x_{2}+3 x_{2}$
Subject to: $x_{1}+2 x_{2}+x_{3}=3$

$$
2 x_{1}-x_{2}=4
$$

$x_{1}, x_{2}, x_{3} \geq 0$

## (4)

## Unit-III

3. (a) Consider the following parametric linear programming problem:

Maximize $\quad z=(10-2 t) x_{1}+(5-3 t) x_{2}$
Subject to: $8 x_{1}+2 x_{2} \leq 48$

$$
2 x_{1}+4 x_{2} \leq 24
$$

$$
x_{1}, x_{2} \geq 0
$$

and $t$ is a non-negative parameter. Perform parametric analysis with respect to the objective function coefficients and identify the ranges of $t$ over which the optimality is unaffected.
(b) Write a short note on interior point algorithm.
(c) Carry out two iterations of Karmarkar's algorithm for the following problem:

Minimize $\quad z=x_{1}-2 x_{2}$
Subject to: $x_{1}-2 x_{2}+x_{3}=0$

$$
\begin{aligned}
& x_{1}+x_{2}+x_{3}=1 \\
& x_{1}, x_{2}, x_{3} \geq 0
\end{aligned}
$$

## (5)

## Unit-IV

4. (a) Discuss the similarities and dissimilarities between Transportation and Assignment problem.
(b) Use Vogel's approximation method to solve the following transportation problem :

|  |  | Destination |  |  |  | Supply |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 1 | 2 | 3 | 4 |  |
| Source | 1 | 3 | 1 | 7 | 4 | 300 |
|  | 2 | 2 | 6 | 5 | 9 | 400 |
|  | 3 | 8 | 3 | 3 | 2 | 500 |
| Demand |  | 250 | 350 | 400 | 200 |  |

(c) Write steps of Hungerian method for solving Assignment problem.

## Unit-V

5. (a) Write steps of PRIM algorithm for finding the Minimum Spanning Tree problem.
(b) A project is composed of 7 activities whose time estimates are listed in the

## ( 6 )

table below. Activities are identified by their beginning ( $i$ ) and ending ( $j$ ) node numbers :

| Activity <br> $(i-j)$ | Estimate <br>  <br> Optimistic <br> $\left(t_{o}\right)$ |  | Most likely <br> $\left(t_{m}\right)$ |
| :---: | :---: | :---: | :---: |
|  | 1 | 1 | 7 |
| $\left(t_{p}\right)$ |  |  |  |$|$| Pessimistic |
| :---: |
| $1-3$ |

(i) Draw the project network.
(ii) Find the expected duration and variance for each activity. What is the expected project length?

## (7)

(c) Consider the following data of the project :

| Activity | Predecessor (s) | Duration <br> (weeks) |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | $t_{o}$ | $t_{m}$ | $t_{p}$ |
| $A$ | - | 3 | 5 | 8 |
| $B$ | - | 6 | 7 | 9 |
| $C$ | - | 4 | 5 | 9 |
| $D$ | $A$ | 3 | 5 | 8 |
| $E$ | $B$ | 4 | 6 | 9 |
| $F$ | $A$ | 5 | 8 | 11 |
| $G$ | $C, D$ | 3 | 6 | 9 |
| $H$ | $C, D, E$ | 1 | 2 | 9 |

(i) Construct the project network.
(ii) Find critical path and expected completion time.

