FD-309

M.A./M.Sc. 1st Semester
Examination, Dec.-Jan., 2021-22

MATHEMATICS

Paper - I

Advanced Abstract Algebra - I

Time : Three Hours] [*Maximum Marks* : 80

Note : Answer any **two** parts from each question. All questions carry equal marks.

1. (*a*) Prove that any two composition series of a finite group are equivalent.

(*b*) Prove that every subgroup of a solvable group is solvable.

(*c*) If G be a nilpotent group, then prove that every subgroup of G and every homomorphic image of G are nilpotent.

2. (*a*) Let $F \leq E \leq K$ be fields. If K is a finite extension of E and E is a finite extension of F, then prove that K is a finite extension of F and

$$[K : F] = [K : E]\,[E : F].$$

(*b*) Prove that, a polynomial of degree n over a field can have at the most n roots in any extension field.

(*c*) Prove that, let F be a field. Then there exists an algebraically closed field K containing F as a subfield .

3. (*a*) Find the degree of splitting field $x^5 - 3x^3 + x^2 - 3$ over Q.

(*b*) Prove that, the prime field of a field F is either isomorphic to Q or to $z/(p)$, p is prime.

(*c*) If α, β be the algebric elements over a field F of characteristic zero, then prove that $F(\alpha, \beta)$ is a simple extension of F.

4. (*a*) Prove that, the set Aut(K) of all automorphisms of a field K form a group under composition of mappings.

(*b*) State and prove fundamental theorem of Galois theory.

(*c*) Find the Galois group of $x^3 - 2 \in Q[x]$.

5. (*a*) Prove that, any quartic over F is solvable by radicals.

(*b*) Show that the polynomial $2x^5 - 5x^4 + 5$ is not solvable by radicals.

(*c*) Let E be the splitting field of $x^n - a \in F(x)$. Then prove that $G(E/F)$ is a solvable group.

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