FD-310

M.A./M.Sc. 1st Semester
Examination, Dec.-Jan., 2021-22

MATHEMATICS

Paper - II

Real Analysis - I

Time : Three Hours] [*Maximum Marks* : 80
[*Minimum Pass Marks* : 16

Note : Answer any **two** parts from each question. All questions carry equal marks.

Unit-I

1. (*a*) State and prove Cauchy's general principle of uniform convergences.

(*b*) Let α be monotonically increasing on $[a, b]$. Suppose $f_n \in R(\alpha)$ on $[a, b]$ for $n = 1, 2, 3, ...$ and suppose $f_n \to f$ uniformly on $[a, b]$. Then prove that $f \in R(\alpha)$ on $[a, b]$ and $\int_a^b f d\alpha = \lim_{n\to\infty} \int_a^b f_n d\alpha$.

(*c*) Show that the series $\sin x + \frac{1}{2}\sin 2x + \frac{1}{3}\sin 3x + ...$ converges uniformly in $0 < a \le x \le b < 2\pi$.

Unit-II

2. (*a*) If $\sum a_n$ is a series of conplex number which converges absolutely, then prove that every rearrangement of $\sum a_n$ converges and they all converges to the same sum.

(*b*) State and prove the converse of Abel's theorem.

(*c*) Find the radius of convergence of the following series :

(*i*) $1 + x + \frac{x^2}{\lfloor 2} + \frac{x^3}{\lfloor 3} + \frac{x^4}{\lfloor 4} + ...$

(*ii*) $1 + 2x + 3x^2 + 4x^3 + ...$

Unit-III

3. (*a*) Let Ω be the set of all invertible linear operators on R^n.

(*i*) If $A \in \Omega$, $B \in L(R^n)$ and $\| B - A \| \| A^{-1} \| < 1$, then prove that $B \in \Omega$.

(*ii*) Ω is open subset. Is $L(R^n)$ and mapping $f : \Omega \to \Omega$ defined by $f(A) = A^{-1}$ for all $A \in \Omega$ is continuous ?

(*b*) State and prove the Taylor's theorem.

(*c*) State and prove the chain rule.

Unit-IV

4. (*a*) Determine the maximum and minimum values of the function

$$f(x, y) = x^2 + y^2 + \frac{3\sqrt{3}}{2} xy$$

subject to the constraint $4x^2 + y^2 = 1$.

(*b*) Prove that of all rectangular parallelopipeds of the same volume the cube has the least surface.

(*c*) If $u = \frac{x+y}{z}, v = \frac{y+z}{x}, w = \frac{y(x+y+z)}{xz}$,

show that *u*, *v*, *w* are not independent and find the relations among them.

Unit-V

5. (*a*) Write the definition of the following :

(*i*) The integral of 1-form

(*ii*) The integral of 2-form

(*iii*) The Triple integral

(*b*) State and prove the partitions of unity.

(*c*) Write a short note on differential form.