FD-311

M.A/M.Sc. 1st Semester
Examination, Dec.-Jan., 2021-22

MATHEMATICS

Paper - III

Topology-I

Time : Three Hours] [*Maximum Marks* : 80

Note : Answer any **two** parts from each question. All questions carry equal marks.

Unit-I

1. (*a*) State and prove Schroeder-Bernstein theorem.

(*b*) Give an example of a topological space different from the discrete and indiscrete spaces in which open sets are exactly the same as closed sets.

(*c*) Let X be a topological space, and let $A \subset X$. Then prove that A is closed iff $D(A) \subset A$.

Unit-II

2. (*a*) Let X be any set and let $\wp$ be the Kuratowski closure operator on X. Then prove that there exists a unique topology τ on X such that for each $A \subset X$, $\wp(A)$ coincides with τ-closure of A.

(*b*) Prove that homeomorphism is an equivalence relation in the collection of all topological spaces.

(*c*) Prove that the property of being a Lindelöf space is a topological property.

Unit-III

3. (*a*) Prove that every subspace of a Hausdorff space is Hausdorff.

(*b*) Prove that a topological space X is normal iff for any closed set F and open set G containing F, there exists an open set V such that

$$F \subset V \subset \overline{V} \subset G$$

(*c*) State and prove Tietze's extension theorem.

Unit-IV

4. (*a*) Prove that a subset A of R is compact iff A is closed and bounded.

(*b*) Prove that a topological space is countably compact iff every countable collection of closed subsets of X with FIP has non-empty intersection.

(*c*) Let (X^*, τ^*) be a one point compactification of a non-compact topological space (X, τ). Then prove that (X^*, τ^*) is Hausdorff iff (X, τ) is Hausdorff and locally compact.

Unit-V

5. (*a*) Let (X, d) be a metric space. Then prove that the following statements are equivalent :

(*i*) X is compact

(*ii*) X is countably compact

(*iii*) X has BWP

(*iv*) X is sequentially compact

(*b*) Prove that a topological space X is disconnected iff there exists a non-empty proper subset of X which is both open and closed in X.

(*c*) Prove that every component of a locally connected space is an open set.