FD-312

M.A./M.Sc. 1st Semester
Examination, Dec.-Jan., 2021-22

MATHEMATICS

Paper - IV

Advanced Complex Analysis-I

Time : Three Hours] [*Maximum Marks* : 80
[*Minimum Pass Marks* : 16

Note : Answer any **two** parts from each questions. All questions carry equal marks.

Unit-I

1. (*a*) State and prove Cauchy's Integral formula.

(*b*) Prove that the function $\sin\left[c\left(z+\frac{1}{z}\right)\right]$ can be expanded in a series of the types $\sum_{n=0}^{\infty} a_n z^n + \sum_{n=1}^{\infty} b_n z^{-n}$ in which the coefficients of both z^n and z^{-n} are :

$$\frac{1}{2\pi}\int_0^{2\pi} \frac{\left[\sin(2c\cos\theta)\cos n\theta\right]}{\sin(2c\cos\theta)\cos n\theta}\, d\theta$$

(*c*) Define Entire function. Find the singularity of the function $\frac{e^{c/(z-a)}}{e^{z/a}-1}$, indicating the characters of each singularity.

Unit-II

2. (*a*) Prove that all the roots of $x^7 - 5z^3 + 12 = 0$ between the circles $|z| = 1$ and $|z| = 2$.

(*b*) State and prove maximum modulus principle.

(*c*) State and prove Inverse function theorem.

Unit-III

3. (*a*) Apply the calculus of residue to prove that :

$$\int_0^{\infty} \frac{dx}{1+x^2} = \frac{\pi}{2}$$

(*b*) Show that :

$$\int_0^{\pi} \frac{\cos 2\theta}{1-2a\cos\theta + a^2} d\theta = \frac{\pi a^2}{1-a^2} \left(a^2 < 1\right)$$

(*c*) Prove by contour integration :

$$\int_0^{\infty} \frac{\log\left(1+x^2\right)}{1+x^2} dx = \pi \log 2$$

Unit-IV

4. (*a*) Show that the transformation $w = \frac{2z+3}{z-4}$ maps the circle $x^2 + y^2 - 4x = 0$ onto the straight line $4u + 3 = 0$ and explain why the curve obtained is not a circle.

(*b*) Let $f(z)$ be analytic function of z in a region D of the z-plane and $f'(z) \neq 0$ inside D. Then prove that the mapping $w = f(z)$ is conformal at the point of D.

(*c*) Discuss the transformation $w = \tan z$.

Unit-V

5. (*a*) Show that, A family F of holomorphic function defined in a domain D, that is $F \subset H(D)$ is normal iff F is locally bounded.

(*b*) Let $\{f_n\}$ be a sequence in $H(G)$ and $f \in (G, C)$ such that $f_n \to f$. Then show that f is analytic and $f_n^{(k)} \to f^{(k)}$ for each integer $k \geq 1$.

(*c*) State and prove Riemann mapping theorem.