FD-313

M.A./M.Sc. 1st Semester
Examination, Dec.-Jan., 2021-22

MATHEMATICS

Paper - V

Advanced Discrete Mathematics - I

Time : Three Hours] [*Maximum Marks* : 80
[*Minimum Pass Marks* : 16

Note : Answer any **two** parts from each question. All questions carry equal marks.

Unit-I

1. (*a*) Define Tautology. If $H_1, H_2, ..., H_m$ and P imply Q, then prove that $H_1, H_2, ..., H_m$ imply $P \rightarrow Q$.

(*b*) What are the quantifiers ? Explain universal quantifiers and existential quantifiers.

(*c*) Show that :

$$\urcorner(P \wedge Q) \rightarrow \left(\urcorner P \vee \left(\urcorner P \vee Q\right)\right) \Leftrightarrow \left(\urcorner P \vee Q\right)$$

Unit-II

2. (*a*) State and prove basic homomorphism theorem.

(*b*) Define submonoid and prove that for any commutative monoid (M, *) the set of idempotent elements of M forms a submonoid.

(*c*) Define direct product of semigroup. Show that the direct product of any two semigroups is a semigroup.

Unit-III

3. (*a*) Define Distributive lattice. Let (L, *, ⊕) be a distributive lattice. For any $a, b, c \in L$, prove that

$$(a * b = a * c) \wedge (a \oplus b = a \oplus c) \Rightarrow b = c$$

(*b*) State and prove De Morgan's law.

(*c*) Define the following

(*i*) Sublattice

(*ii*) Direct product

(*iii*) Boolean algebra

(*iii*) Lattice as partially order set

Unit-IV

4. (*a*) Use the Karnaugh map representation to find a minimal sum-of-product of the following function :

$$f(a, b, c, d) = \Sigma(10, 12, 13, 14, 15)$$

(*b*) Define the following :

(*i*) Join-irreducible

(*ii*) Atoms and Minterms

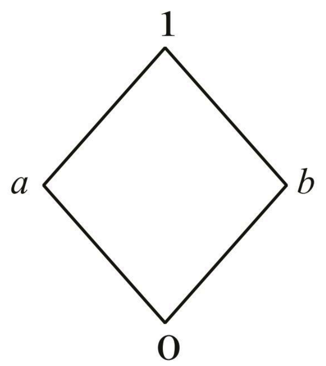
(*iii*) Gates

(*iv*) Canonical forms

(*c*) Find the value of

$$x_1 * x_2 \left[(x_1 * x_4) \oplus x_2' \oplus (x_3 * x_1') \right]$$

for $x_1 = a$, $x_2 = 1$, $x_3 = b$ and $x_4 = 1$, where $a, b, 1 \in B$ and the Boolean algebra $(B, *, \oplus, ', 0, 1)$ is shown in the following figure :



Unit-V

5. (*a*) Define Polish notation; prove that the rank of any well formed polish formula is 1 and the rank of any proper head of a polish is greater than or equal to 1.

(*b*) State and prove Pumping Lemma.

(*c*) Define grammar and let language

$L(Gs) = \{a^n b^n c^n \mid n \geq 1\}$ is generated by

the following grammar

$$Gs = \langle \{S, B, C\}, \{a, b, c\} S, \phi \rangle$$

where ϕ consists of the production
$S \to aSBC$, $S \to aBC$, $CB \to BC$,
$aB \to ab$, $bB \to bb$ $bC \to bc$, $cC \to cc$
then find the derivation for the strings
abc, $a^2b^2c^2$ and $a^3b^3c^3$.