FD-612

M.A./M.Sc. 3rd Semester
Examination, Dec.-Jan., 2021-22

MATHEMATICS

Paper - I

Integration Theory and Functional Analysis

Time : Three Hours] [*Maximum Marks* : 80
[*Minimum Pass Marks* : 16

Note : Answer any **two** parts from each question. All questions carry equal marks.

Unit-I

1. (*a*) If E is a measurable set of finite positive measure, i. e., $0 < \nu(E) < \infty$, then prove that E contains a positive set A with $\nu(A) > 0$.

(*b*) State and prove Lebesgue Decomposition theorem.

(*c*) State and prove Caratheodory Extension theorem.

Unit-II

2. (*a*) If A is a μ-measurable subset of X and B is a ν-measurable subset of Y, then prove that $A \times B$ is a $\mu \times \nu$-measurable subset of $X \times Y$.

(*b*) Let E be a set in $R_{\sigma\delta}$ with $(\mu \times \nu)(E) < \infty$. Then show that the function g defined by

$$g(x) = \nu(E_x)$$

is a measurable function of x and

$$\int g \, d\mu = (\mu \times \nu)(E).$$

(*c*) Prove that every finite signed Borel measure μ on R^k that is absolutely continuous with respect to the Lebesgue measure λ, is differentiable almost everywhere.

Unit-III

3. (*a*) Prove that every compact Baire set is a G_δ.

(*b*) Let μ be a measure defined on a σ-algebra }}} containing the Baire sets. If μ is quasi regular, then prove that for each $E \in$ }}} with $\mu(E) < \infty$ there is a Baire set B with

$$\mu(E \Delta B) = 0$$

(*c*) State and prove Riesz-Markoff theorem.

Unit-IV

4. (*a*) Let X be a non-zero finite-dimensional linear space of dimension n. If X is complete, then show that it is isomorphic to C^n.

(*b*) Show that on a finite dimensional linear space all norms are equivalent.

(*c*) Show that a normed linear space X is complete if and only if every absolutely convergent series in X is convergent.

Unit-V

5. (*a*) Prove that in a normed linear space X,

$x_n \xrightarrow{w} x$ if and only if :

(*i*) The sequence $\{\| x_n \|\}$ is bounded.

(*ii*) For every element f of a total subset $M \subset X^*$, $f(x_n) \to f(x)$.

(*b*) Let X and Y be normed linear spaces and T a linear transformation on X into Y. Then T is continuous either at every point of X or at no point of X. It is continuous on X if and only if there is a constant M such that $\| Tx \| \leq M \cdot \| x \|$ for every x in X.

(*c*) Show that the dual space of c_0 is l_1.
