FD-613

M.A/M.Sc. 3rd Semester
Examination, Dec.-Jan., 2021-22

MATHEMATICS

Paper - II

Partial Differential Equations
and Mechanics - I

Time : Three Hours] [*Maximum Marks* : 80

Note : Answer any **two** parts from each question. All questions carry equal marks.

Unit-I

1. (*a*) Solve the partial differential equation $p + r + s = 1$

(*b*) If ϕ is harmonic function in R_1 and $\frac{\partial \phi}{\partial n} = 0$ on R_2, then ϕ is a constant in $\bar{R}$.

(*c*) Find the Green's function for the Dirichlet problem on the rectangle $R_1 : 0 \le x \le a,\ 0 \le y \le b$, described by the PDE.

$$(\Delta^2 + \lambda)u = 0 \text{ in } R_1$$

and the BC, $u = 0$ on R_2

Unit-II

2. (*a*) State and prove Mean value theorem for Harmonic function.

(*b*) Derive the one dimensional wave equation.

(*c*) Obtain the solution of the heat flow equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ by the method of separation of variables.

Unit-III

3. (*a*) State and prove Lagrange's equation of first kind.

(*b*) Derive the Hamilton canonical equations.

(*c*) Derive Ruth's equation.

Unit-IV

4. (*a*) Define Poisson bracket. If [ϕ, ψ] be the Poisson bracket of ϕ and ψ, then prove that :

(*i*) $\frac{\partial}{\partial t}[\phi,\psi]=\left[\frac{\partial\phi}{\partial t},\psi\right]+\left[\phi,\frac{\partial\psi}{\partial t}\right]$

(*ii*) $\frac{d}{dt}[\phi,\psi]=\left[\frac{d\phi}{dt},\psi\right]+\left[\phi,\frac{d\psi}{dt}\right]$

(*b*) Find a curve joining two points along with a particle falling from rest under the influence of gravity travels from higher to the lower point in the minimum time.

(*c*) Show that the transformation :

$$P=\frac{1}{2}\left(p^2+q^2\right),\ Q=\tan^{-1}\left(\frac{q}{p}\right)$$

is canonical.

Unit-V

5. (*a*) Find the attraction of thin spherical shell of mean M and radius a.

(*b*) Show that the potential of a uniform spherical shell, of small thickness *k*, density ρ and radius *a* at an external point-distant *c* from the centre is

$$\frac{2\pi\gamma k\rho a}{(n+1)(n+3)c}\left[(c+a)^{n+2}-(c-a)^{n+3}\right]$$

(*c*) State and prove Gauss' theorem.