FD-616

M.A./M.Sc. 3rd Semester
Examination, Dec.-Jan., 2021-22

MATHEMATICS

Optional (C)

Paper - III

Fuzzy Sets and its Applications - I

Time : Three Hours] [*Maximum Marks* : 80
[*Minimum Pass Marks* : 16

Note : Answer any **two** parts from each question. All questions carry equal marks.

1. (*a*) Define law of excluded middle and law of contradiction and discuss the distributive property of (i, u, c) which satisfies these two laws.

(*b*) State characterization theorem of *t*-conorms and find *t*-conorm for $g(a) = 1 - (1 - a)^p$.

(*c*) Define convexity for a set graphically and show that a Fuzzy set A on R is convex iff

$A(\lambda x_1 + (1-\lambda)x_2) \geq \min [A(x_1), A(x_2)]$.

2. (*a*) Explain extension principle, how it differs from crisp function. Show that $\alpha [f(A)] \geq f(\alpha_A)$. Give a supportive example.

(*b*) Solve Fuzzy equation $A + X = B$ where

$$A = \frac{\cdot 3}{[0,1)} + \frac{\cdot 5}{[1,2)} + \frac{\cdot 8}{[2,3)} + \frac{\cdot 9}{[3,4)} + \frac{1}{4} + \frac{\cdot 6}{(4,5]} + \frac{\cdot 2}{(5,6]}$$

$$B = \frac{\cdot 2}{[0,1)} + \frac{\cdot 3}{[1,2)} + \frac{\cdot 6}{[2,3)} + \frac{\cdot 5}{[3,4)} + \frac{\cdot 8}{[4,5)} + \frac{1}{6} + \frac{\cdot 5}{(6,7]} + \frac{\cdot 4}{(7,8]} + \frac{\cdot 2}{(8,9]} + \frac{\cdot 1}{(9,10]}$$

(*c*) $$A(x) = \begin{cases} 0 & \text{for } x < -2 \text{ and } x > 4 \\ \dfrac{x+2}{3} & \text{for } -2 \leq x \leq 1 \\ \dfrac{4-x}{3} & \text{for } 1 \leq x \leq 4 \end{cases}$$

$$B(x) = \begin{cases} 0 & \text{for } x < 1 \text{ and } x > 3 \\ x - 1 & \text{for } 1 \le x \le 2 \\ 3 - x & \text{for } 2 \le x \le 3 \end{cases}$$

Find
MIN $(A, B)(x)$ and MAX $(A, B)(x)$.

3. (*a*) Define crisp and fuzzy relations. Let $X = \{1, 2, ..., 10\}$. The cartesian product $(x \times y)$ contains 100 members. Let $R(X, X) = \{(x, y) \mid x \text{ and } y \text{ have the same remainder when divided by } 3\}$. Is R an equivalance relation on X? Find equivalance classes.

(*b*) Write a short note on Fuzzy morphisms.

(*c*) Prove that

(*i*) $w_i(a, d) \ge b$ iff $i(a, b) \le d$

(*ii*) $w_i(\inf a_j, b) \ge \sup w_i(a_j, b)$

4. (*a*) Let $X = \{1, 2, ..., 100\}$, $Y = \{50, 51, ...,100\}$

$$R(X,Y) = \begin{cases} 1 - \dfrac{x}{y} & x \le y \\ 0 & \text{otherwise} \end{cases}$$

(*i*) What is the domain of R?

(*ii*) What is the range of R?

(*iii*) Calculate R^{-1}

(*b*) Prove that min join are associative operations on binary fuzzy relations.

(*c*) Write a short note on fuzzy compatibility relations.

5. (*a*) Define the following :

(*i*) Total ignorance

(*ii*) Fuzzy measure

(*iii*) Degree of belief

(*iv*) Necessity measure

(*b*) If $X = \{a, b, c, d\}$, $m_1(a, b) = \cdot 2$, $m_1(a, c) = \cdot 3$, $m_1(b, d) = \cdot 5$, $m_2(a, d) = \cdot 2$, $m_2(b, c) = \cdot 5$, $m_2(a, b, c) = \cdot 3$. Calculate the basic probability assignment.

(*c*) $F = \frac{\cdot 4}{1} + \frac{\cdot 7}{2} + \frac{1}{3} + \frac{\cdot 8}{4} + \frac{\cdot 5}{5}$ and $A(x) = 0$ for all $x \notin \{1, 2, 3, 4, 5\}$. Determine Nec ($A$) and Pos ($A$).

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