# FD-621

M.A/M.Sc. 3rd Semester
Examination, Dec.-Jan., 2021-22

## MATHEMATICS

Optional (B)

Paper - V

Graph Theory-I

*Time* : Three Hours] [*Maximum Marks* : 80

**Note** : Answer any **two** parts from each question. All questions carry equal marks.

**1.** (*a*) Prove that if a graph $H$ is homeomorphic from a graph $G$, then $G$ is a contraction of $H$.

(*b*) Prove that any homomorphism is the product of a connected and a discrete homomorphism.

(*c*) Prove that a graph $G$ is contractible to a graph $H$ and $\Delta(H) \leq 3$. Then $G$ has a subgraph homeomorphism from $H$.

**2.** (*a*) Prove that if $G$ is connected and has diameter $d$ then the adjacency algebra has dimension atleast $d + 1$.

(*b*) Prove that any square submatrix of the adjacency matrix $F$ of a graph $G$ has determinant $+1, -1$ or zero.

(*c*) Prove the sum of any two cuts of a graph $G$ is also a cut of $G$.

**3.** (*a*) Prove that if a connected k-chromatic graph has exactly one vertex of degree exceeding $k$-1 then it is minimal.

(*b*) Prove that any uniquely $k$-colorable graph is ($k$-1) connected.

(*c*) Prove that every planar graph is 5-vertex colorable.

**4.** (*a*) Prove that for any graph $G$, $\alpha_0 + \beta_0 = n$.

(*b*) Prove that for any graph $G$ of order $n \geq 2$ without isolated vertices $\pi_1 \leq \left[\frac{n^2}{4}\right]$ and the partition need use only edges and triangles.

(*c*) Prove that for any connected graph $G$, $n \geq 2\beta_0 - 1$.

**5.** (*a*) Prove that a graph is triangulated iff every minimal vertex-separator induces a complete subgraph.

(*b*) Prove that every strongly perfect graph is perfect.

(*c*) Prove that a graph $G$ is a permutation graph iff $G$ and $\bar{G}$ are comparability graphs.

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