FD-763

M.A./M.Sc. 4th Semester
Examination, May-June 2022

MATHEMATICS

Paper - II

Partial Differential Equations and Mechanics

Time : Three Hours] [*Maximum Marks* : 80

Note : Answer any **two** parts from each question. All questions carry equal marks.

Unit-I

1. (*a*) State and prove characteristic ODE.

(*b*) State and prove Lax-Oleinik formula.

(*c*) State and prove convex duality of Hamilton and Lagrangian.

Unit-II

2. (*a*) Derive Barenblatt's solution of porous medium equation.

(*b*) Prove that for Fourier Transform

(*i*) $\widehat{D^{\alpha}u} = (iy)^{\alpha} \cdot \hat{u}$

(*ii*) $(u * v)^{\wedge} = (2\pi)^{\frac{n}{2}} \hat{u}\,\hat{v}$

(*c*) Write short notes on Hodograph and Legendre Transform.

Unit-III

3. (*a*) Write about the following :

(*i*) Singular perturbations

(*ii*) Geometric optics

(*b*) State and prove Cauchy-Kovalevskaya theorem.

(*c*) Write about the following :

(*i*) Homogenations

(*ii*) Stationary phase for the wave equation

Unit-IV

4. (*a*) State and prove Whittaker equation.

(*b*) State and prove Lee Hwa Chung theorem.

(*c*) State and prove Hamilton principle for conservative system.

Unit-V

5. (*a*) Prove that the Lagrange Bracket is invariant under canonical transformation.

(*b*) Prove that :

(*i*) $\left[q_j, p_k\right] = \delta_{jk}$

(*ii*) $\left[q_j, q_k\right] = 0$ for Poisson Bracket

(*c*) Prove that :

$$\sum_{l=1}^{2n} \{u_l, u_i\} \cdot \left[u_l, u_j\right] = \delta_{ij}$$

for Lagrangian and Poisson Brackets.