Roll No. ....

DD-461

M. A./M. Sc. (Second Semester) EXAMINATION, May-June, 2020

MATHEMATICS

Paper Third

(General and Algebraic Topology)

Time : Three Hours

Maximum Marks : 80

Note : Attempt any *two* parts from each Unit. All questions carry equal marks.

Unit—I

1. (a) Define Tychonoff product topology in terms of standard subbase.

(b) Let $f : A \to \prod_{\alpha \in \Lambda} X_\alpha$ be given by the equation :

$$f(a) = (f_\alpha(a))_{\alpha \in \Lambda}$$

where $f_\alpha : A \to X_\alpha$ for each α. Let $\prod X_\alpha$ have the product topology. Then prove that f is continuous iff each function f_α is continuous.

(c) If each space X_α is Hausdorff space, then prove that $\prod X_\alpha$ is also Hausdorff.

Unit—II

2. (a) If $\{(X_\alpha, \tau_\alpha)\}_{\alpha\in\Lambda}$ is a family of topological spaces and $\left(X = \prod_{\alpha\in\Lambda} X_\alpha, \tau\right)$ is its product space, then prove that if (X, τ) is normal, then so is (X_α, τ_α) for each $\alpha \in \Lambda$.

(b) If $\left(X = \prod_{\alpha\in\Lambda} X_\alpha, \tau\right)$ is the product space of topological spaces $\{(X_\alpha, \tau_\alpha)\}_{\alpha\in\Lambda}$ and (X, τ) is connected, then prove that each (X_α, τ_α) is connected.

(c) Prove that a countable product of second countable space is second countable.

Unit—III

3. (a) State and prove Embedding Lemma.

(b) Prove that every subcollection of a locally finite space is locally finite.

(c) State and prove the Smirnov Metrization theorem.

Unit—IV

4. (a) Let (X, τ) be a topological space and $Y \subset X$. If x_0 is a point of X, then prove that $x_0 \in \overline{Y}$ iff $\exists a$ net in Y converging to x_0.

(b) Let **F** be a filter on a non-empty set X and let $A \subset X$. Then prove that $\exists a$ filter **F′** finer than **F** such that $A \in$ **F′** iff $A \cap B \neq \phi$ for every $B \in$ **F**.

(c) Prove that a filter F on X is an ultrafilter iff F contains all those subsets of X which intersect every member of F.

Unit—V

5. (a) Prove that the relation '$\simeq$' of homotopy is an equivalence relation.

(b) Prove that in a simply connected space X, any two paths having the same initial and final points are path homotopic.

(c) Prove that the fundamental group of S' is isomorphic to the additive gorup of integers.