

**DD-771**

**M. A./M. Sc. (Fourth Semester)  
EXAMINATION, 2020**

**MATHEMATICS**

**(Optional—B)**

**Paper Fifth**

**(Graph Theory)**

*Time : Three Hours*

*Maximum Marks : 80*

**Note :** Attempt any *two* parts from each question. All questions carry equal marks.

**Unit—I**

1. (a) For any  $s, t \geq 1$ , prove that there is  $R(s, t) < \infty$  such that any graph on  $R(s, t)$  vertices contains either an independent set of size  $s$  or a clique of size  $t$ . In particular :

$$R(s, t) \leq \binom{s+t-2}{s-1}$$

**P. T. O.**

- (b) Prove that every graph on  $\binom{k+l}{k}$  vertices contains either a complete sub-group on  $k+1$  vertices or an independent set of  $l+1$  vertices.
- (c) Prove that for any  $s \geq 2$ ,

$$R(s, s) \geq 2^{\frac{s}{2}}$$

## Unit—II

2. (a) Prove that every vertex of a composite connected graph lies on a 4-cycle.
- (b) Prove that the line-group and the point group of a graph  $G$  are isomorphic iff  $G$  has at most one isolated point and  $k_2$  is not a component of  $G$ .
- (c) Prove that the group of the union of two graphs is the sum of their groups :

$$\Gamma(G \cup G_2) \equiv \Gamma(G_1) + \Gamma(G_2)$$

iff no component of  $G$ , is isomorphic with a component of  $G_2$ .

## Unit—III

3. (a) Prove that every 3-chromatic maximal planar graph is uniquely 3-colorable.
- (b) If  $B_1, B_2, \dots, B_r$  are the blocks of a graph  $G$ , then prove that :

$$\phi(G, x) = \frac{1}{x^{r-1}} \prod_{i=1}^r Q(B_i, x).$$

- (c) Prove that the coefficients of every chromatic polynomial alternate in sign.

## Unit—IV

4. (a) Prove that each cycle  $C_n, n \geq 3$  is chromatically unique.

- (b) Prove that for any graph  $G$  :

$$\chi(G) \leq 1 + \max \delta(G'),$$

where the maximum is taken over all induced subgroups  $G'$  of  $G$ .

- (c) Prove that for a connected graph  $G$ ,

$$T(G, 1, 1) = \tau(G),$$

the number of spanning trees of  $G$ .

## Unit—V

5. (a) Prove that a graph  $G$  is  $n$ -line connected iff every pair of points are joined by at least  $n$ -line disjoint paths.
- (b) State and prove Menger's theorem for digraph (vertex form).
- (c) Prove that every acyclic graph without isolated has a unique basis consisting of its transmitters.