Roll No. .....

DD-463

M. A./M. Sc. (Second Semester) EXAMINATION, May-June, 2020

MATHEMATICS

Paper Fifth

(Advanced Discrete Mathematics—II)

Time : Three Hours

Maximum Marks : 80

Note : Attempt all the *five* questions selecting *two* parts from each Unit. All questions carry equal marks.

Unit—I

1. (a) Define graph and prove that the number of vertices of odd degree in a graph is always even.

   (b) Define the following :

   (i) Sub-graph

   (ii) Degree of a vertex

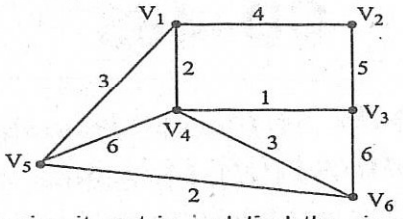
   (iii) Complete graph

   (iv) Planar graph

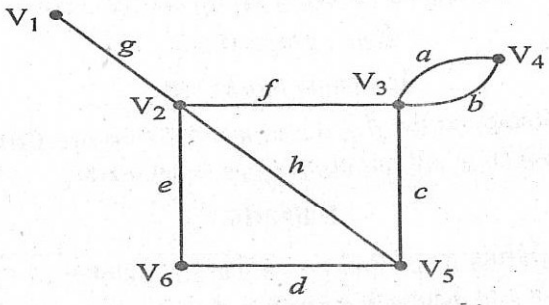
   (c) Show that a simple graph with n vertices and k components can have at most $\frac{(n-k)(n-k+1)}{2}$ edges.

Unit—II

2. (a) Define tree and find the minimal spanning tree for the weighted graph in the following figure using Kruskal's algorithm :



(b) Define circuit matrix and find the circuit matrix in given graph :



(c) Define spanning tree and prove that a tree with n vertices has $n - 1$ edges.

Unit—III

3. (a) Explain Dijkstra's algorithm in brief.

(b) Write short notes on the following :

(i) In degree and out degree of a vertex

(ii) Directed tree and search trees

(c) Define the following :

(i) Directed graph

(ii) Weighted undirected graph

(iii) Strong connectivity

(iv) Tree traversals

Unit—IV

4. (a) Minimize finite state machine M, where M is given by the following state table :

State	Input		Output
	0	1	
→ s_0	s_3	s_1	0
s_1	s_4	s_1	1
s_2	s_3	s_0	0
s_3	s_2	s_3	1
s_4	s_1	s_0	0

(b) Define transition system. Consider the finite state machine where transition function δ is given by the following table in the form of a transition table. Here, $Q=\{q_0, q_1, q_2, q_3\}$, $\Sigma=\{0,1\}$, $F=\{q_0\}$. Give the entire sequence of states for the input string 1011011 :

Transition Function Table

State	Input	
	0	1
→ (q_0)	q_2	q_1
q_1	q_3	q_0
q_2	q_0	q_3
q_3	q_1	q_2

(c) Write short notes on the following :

(i) Finite state machines and their transition table diagrams

(ii) Reduced machine and Homomorphism

Unit—V

5. (a) Define non-deterministic finite automata and find a deterministic acceptor equivalent to $M=(\{q_0, q_1, q_2\}, \{a, b\}, \delta, q_0, \{q_2\})$ δ is given in the following table :

State Table

States/Σ	a	b
$\rightarrow$ (q_0)	q_0, q_1	q_2
q_1	q_0	q_1
q_2		q_0, q_1

(b) Define Mealy machine and consider the Moore machine described by the transition table given by table. Construct the corresponding Mealy machine :

Moore Machine

Present State	Next State		Output
	$a=0$	$a=1$	
$\rightarrow q_1$	q_1	q_2	0
q_2	q_1	q_3	0
q_3	q_1	q_3	1

(c) Define the following :

(i) DFA

(ii) Moore Machine

(iii) Finite Automata

(iv) Acceptors